Broadening Student Knowledge of Dynamics by Means of Simulation Software

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Introduction

- The wide availability of commercial mathematical software, such as MAPLE®, has made it possible to expand student knowledge of mechanics.
- New problems, usually non-linear, can be introduced which previously could not be treated because of the lack of analytic solutions.
- Note that the thrust here is not to debate the merit of finite difference schemes in different packages.
- By means of numerical solutions, students can get a feel for finite difference approaches and, more importantly, their physical understanding can be enhanced and new phenomena explored.
- An assumption is that students have been exposed to differential equations (either as a pre or co-requisite).
- Examples are given with non-dimensional equations → minimal commitment to numerical values.

Effect of Viscous Damping on the Stability of an Inverted Pendulum

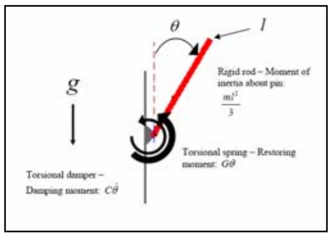
- Consider an inverted pendulum with a torsional spring and torsional damper at the base
- Equation of motion:

$$ml^2\ddot{\theta} + C\dot{\theta} + G\theta - mg\frac{l}{2}\sin(\theta) = 0$$

- Dimensionless form:

$$\frac{d^2\theta}{d\tau^2} + J\frac{d\theta}{d\tau} + B\theta - \sin(\theta) = 0$$

- For unstable states, the equation of motion can be solved (using MAPLE®) with initial conditions close to the equilibrium states.

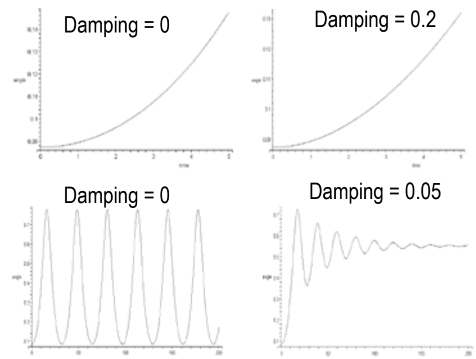


Linear Model

- Unstable response
- No effect of damping
- Unrealistic results

Non-Linear Model

- Bounded response
- Damping causes a faster approach to steady state
- Realistic results



Key Points

- As the response grows the linear model breaks down, and the non-linear model must be used.
- The predictions are radically different!
- The first state (θ=0) is unstable, and the system migrates to the second state (θ= 31.62°).

Harmonic Motion of a Hardening Spring-Damper-Mass System

- Equation of motion:

$$\frac{d^2x}{dt^2} + 2\beta\omega_0\frac{dx}{dt} + \omega_0^2 x + \frac{k_1}{m}x^3 = \frac{Q}{m}\sin(\omega t)$$

- Dimensionless form:

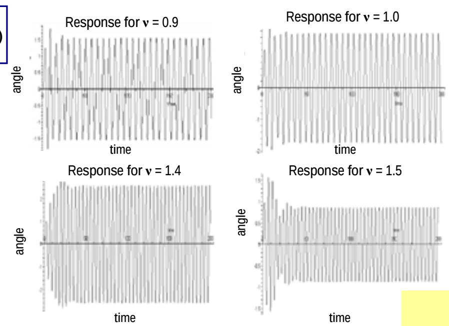
$$\frac{d^2\chi}{d\tau^2} + 2\beta\frac{d\chi}{d\tau} + \chi + \delta\chi^3 = \sin(\nu t)$$

- Numerical solution (using MAPLE®)
- Weak non-linearity

$$\delta = 0.25$$

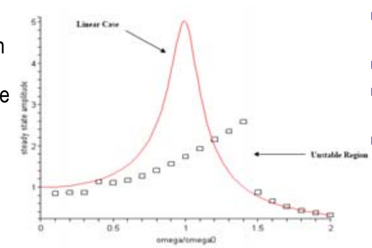
- Light damping

$$\beta = 0.10$$



Results

- With increasing ν, there is an increase of the steady state values and then a sudden drop ("jump") at ν= 1.5
- Students should notice the "bending" of the resonant peak in the case of a non-linear system. Here the peak is bent to the right since the non-linearity is of a hardening type.
- Students should note that unstable solutions exist in the vicinity of ν =1.5.



Key Points

- In the vicinity of resonance the linear spring model breaks down.
- A non-linear model must be used.
- Resonance does not occur at the predicted linear resonance frequency!
- A new phenomena occurs ("jump") close to the resonance frequency → unstable solutions exist.

Conclusions

- In most undergraduate engineering courses students are introduced to mathematics software such as MAPLE®.
- For dynamics courses, some intractable problems can then be explored in order to demonstrate interesting and important physical phenomena.
- The examples presented here were:
  - (i) The effect of viscous damping on the stability of an inverted pendulum. It was shown that with a linear model viscous damping does not stabilize an unstable state, whereas, damping plays an important role when a non-linear model is considered.
  - (ii) Forced harmonic motion of a non-linear hardening spring-mass system. The numerical simulation of the response illustrates a "jump phenomena" in which the steady state amplitude undergoes a jump in passing through frequencies close to the linear resonance frequency.

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