Teaching Flux in the Age of Desktop Monte Carlo

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Background

- Scalar flux is an important concept in nuclear engineering analysis because it is used to compute reaction rates
- Scalar flux is a pathlength (PL) rate density of particles

$$\dot{P}L = \int_{\Gamma} \phi d^3 r$$

- Monte Carlo codes use the pathlength interpretation to compute flux. Students need to understand this deeply.

Student Confusion

- Students confuse flux with current
  - They mistakenly use flux to "compute" rate of particle flow across surfaces
  - Who can blame them, based on the name, the units, and the elementary examples?
- Students cannot really defend the pathlength rate density interpretation of flux

Path Length Density

Approaches to Teaching Flux

1. Number density times speed $\phi = v \times n$
2. International Commission on Radiation Units definition

$$\phi = \lim_{R \to 0} \frac{\text{rate of particle entry into sphere of radius } R}{\pi R^2}$$

3. As pathlength rate density

- Best flux estimator in Monte Carlo is <u>pathlength estimator</u>

$$\int_{\Gamma} \phi d^3 r \sim \frac{1}{N} \sum_i l_i$$

- Weinberg & Wigner's argument (1958)

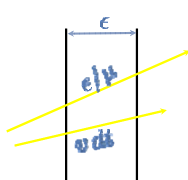
$$\phi d^3 r = v \times n d^3 r$$

Rate at which one particle generates pathlength

Number of particles in the differential box

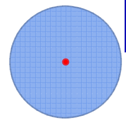
- Surface average flux estimator – what's the right pathlength?

$$\int_A \phi d^2 r = \frac{1}{\varepsilon A N} \sum_i \frac{\varepsilon}{\mu_i} = \frac{1}{AN} \sum_i \frac{1}{\mu_i}$$



Simple Examples

Point Source within the Sphere



$$\dot{P}L = \int \frac{S_o e^{-\Sigma r}}{4\pi r^2} \times 4\pi r^2 \times dr$$

Number of particles that reach r

Pathlength

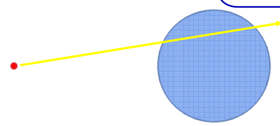
- Use $\frac{1}{4\pi r^2}$ idea and relate to pathlength (PL)
- Requires careful discussion of current vs. flux

Point Source Separated from a Sphere

$$\dot{P}L = \int_0^{\theta_{\max}} 2\sqrt{R^2 - x^2 \sin^2(\theta)} \frac{S}{4\pi} 2\pi \sin(\theta) d\theta$$

Pathlength

Rate particles enter differential core

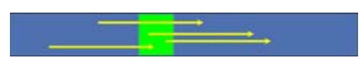


$$\int_{sphere} \frac{S}{4\pi r^2} d^3 r$$

- Compute volume average flux
- Compute total pathlength
- Students discover they are equal!

Path Length in Finite Region

Compute PL generated during time T within long tube of differential cross section by particles in differential solid angle



- Some particles cross the green region
- Some enter but don't leave
- Some are there at the start, and leave
- Some are born in there
- Some are there and don't leave

$$PL = \sum_i^N PL_i$$

Upper and Lower Bounds

Upper Bound

- Ignores interactions and shows that particles in the cell don't contribute in the limit

$$PL_i < \left(T\psi_i + \frac{\psi_{im}}{\upsilon} \Delta l + TQ_{im} \Delta l \right) dA d^2 \Omega \Delta l$$

Lower Bound

- Includes only particles that cross cell and includes interactions

$$PL_i > (T - \Delta l/\upsilon)(1 - e^{-\Sigma_i \Delta l}) \psi_i dA d^2 \Omega \Delta l$$

All particles entering in this time cross the cell

Over-estimate of probability of particle loss

Path Length and Flux

$$PL = \lim_{N \to \infty} \sum_i PL_i$$

This is a volume integral

$$PL = T \int_0^L \psi(r_o + l\Omega, E, \Omega) dl dA d^2 \Omega$$

- Pathlength (PL) comes only from particles that enter dl
- Volumetric effects in dl generate PL only in later regions

Conclusion

- Because the best estimator of flux in Monte Carlo codes is based on understanding the volume integral of scalar flux as the rate of generation of particle pathlength, students must be taught this idea, and learn it well.
- Examples, from simple to complex, exist to show the connection between pathlength and flux.
- Examples that include interactions are important for student understanding
- The confusion between "flux" and current must be further addressed in our curricula

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